8.2 l'hopital's rule

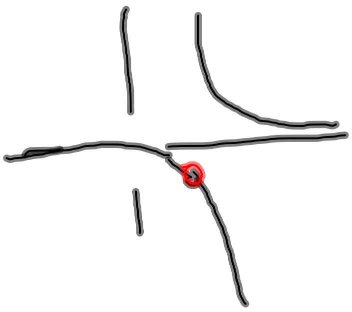
example

$$\lim_{x\to 2} \underbrace{\frac{\sin(x-2)}{(x-2)^2 - \frac{1}{2}(x-2)}}_{h(x)}$$

subst. let $x=2$ get $\frac{0}{0}$

$h(2)$ undef.

the limit is defined



hole.

limit = y-coord

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why does l'hopital's rule work?

$$\lim_{x\to 2} \frac{f(x)}{g(x)} = \lim_{x\to 2} \frac{f'(2)(\cancel{x-2}) + \overset{0}{f(2)}}{g'(2)(\cancel{x-2}) + \overset{0}{g(2)}}$$

replaced $f(x)$ & $g(x)$ with their tan lines

$f(2)=0$

$g(2)=0$

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l'hopital's rule

if $f(a)=0,\ g(a)=0$

then $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{f'(a)}{g'(a)}$

stronger version

$$\boxed{\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}}$$

need $\frac{0}{0}$ indeterminate or $\frac{\infty}{\infty}$

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$$\lim_{x\to 0} \frac{\sin x}{x^2} = \lim_{x\to 0} \frac{\cos x \to 1}{2x \to 0} = \infty$$

$$\lim_{x\to \frac{\pi}{2}} \frac{\sec x}{1+\tan x} \qquad \frac{\infty}{\infty}$$

$$\lim_{x\to \frac{\pi}{2}} \frac{\cancel{\sec x}\tan x}{\sec^{\cancel{2}} x}$$

$$\lim_{x\to \frac{\pi}{2}} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \lim_{x\to \frac{\pi}{2}} \sin x = 1$$

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other indeterminates $\infty \cdot 0$, $\infty - \infty$, 1^∞, 0^0, ∞^0

$$\lim_{x\to\infty}\Big(\underbrace{1+\tfrac{1}{x}}_{1}\Big)^{x \to \infty} = e$$

$$\left(1+\frac{1}{x}\right)^x \Big|_{x=10000} = 2.7181\ldots$$

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$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = y \qquad 1^\infty$$

$$\lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x = \ln y$$

$$\lim_{x\to\infty} x\cdot\ln\left(1+\frac{1}{x}\right) = \ln y$$

$$\lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \quad \frac{0}{0} \quad = \ln y$$

can use l'hopitals

$$\lim_{x\to\infty} \frac{\cancel{\left(-\frac{1}{x^2}\right)}\frac{1}{1+\frac{1}{x}}}{\cancel{-\frac{1}{x^2}}} = \ln y \qquad \tfrac{1}{x}\to 0$$

$$1 = \ln y$$

$$e = y$$

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$$\infty - \infty$$

$$\lim_{x\to 1}\left(\frac{1}{\ln x}\cdot\frac{(x-1)}{(x-1)} - \frac{1}{x-1}\cdot\frac{\ln x}{\ln x}\right)$$

$$\lim_{x\to 1}\frac{x-1-\ln x}{\ln x\,(x-1)} \qquad \frac{0}{0}$$

$$\frac{0}{0} \quad \lim_{x\to 1}\frac{\left(1-\frac{1}{x}\right)x}{\left(\ln x\cdot 1 + (x-1)\frac{1}{x}\right)x} = \lim_{x\to 1}\frac{x-1}{x\ln x + (x-1)}$$

$$= \lim_{x\to 1}\frac{1}{x\cdot\frac{1}{x} + \ln x\cdot 1 + 1}$$

$$= \frac{1}{2}$$

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